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SCHEMATIC LEARNING OF INTRODUCTORY CONCEPTS OF ARITHMETIC

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SCHEMATIC LEARNING OF INTRODUCTORY CONCEPTS OF ARITHMETIC

- The main goal of early mathematical activities is primarily focused on the formation of internal representations of mathematical concept.
- Forming the idea of number begins with the perception of a collection of objects followed by an abstraction of their essential properties.
- Number is a primary concept and so basic that it is actually quite hard to say which quality is abstracted — it is easier to note what qualities are ignored as being inessential noise.

SCHEMATIC LEARNING OF INTRODUCTORY CONCEPTS OF ARITHMETIC

- Idea of number is formed when we perceive different collections of objects and when we ignore:

  a) *the nature of the objects*, and

  b) the arrangement and grouping of the objects.
- Grouping of elements is inessential for forming

  number concept, but that noise is meaningful when arithmetic expressions are formed.
- We express two kinds of meaningful noise:

  a) decimal grouping of elements

  b) grouping into additive schemes

SCHEMATIC LEARNING OF INTRODUCTORY CONCEPTS OF ARITHMETIC

- Decimal grouping of elements is noise for abstract number understanding, but it is relevant for its decimal notation.
- Decimal notation is based on grouping that leads to an easy comparison of numbers.
- Another kind of meaningful noise are schemes on which we are reacting when adding, subtracting or generally when composing arithmetic expressions.

SCHEMATIC LEARNING OF INTRODUCTORY CONCEPTS OF ARITHMETIC

- Learning addition and subtraction, children are directed first to perceive *additive schemes*, being two disjoint sets together with their union. These schemes are followed by an *addition task* – the number of elements of the two sets is given and the number of elements of the union is searched or by a *subtraction task* – the number of elements of the union and one of the two sets is given and the number of elements of the other set is searched. Practicing these activities, the meaning of these two operations establishes what has to precede formal calculation tasks.

- In the same way how the representation of a set precedes the idea of its cardinal number, the representation of an additive scheme precedes addition and subtraction, depending on which of the two tasks it is associated with. Didactical realization of the both tasks consists of a display of pictorial representations of pairs of sets. The sets A and B should be chosen so that their elements, as well as the elements of AUB, are named differently (for example (*Fig.1*), red cars, blue cars and cars; (*Fig.2*) orange juices, strawberry juices and juices.

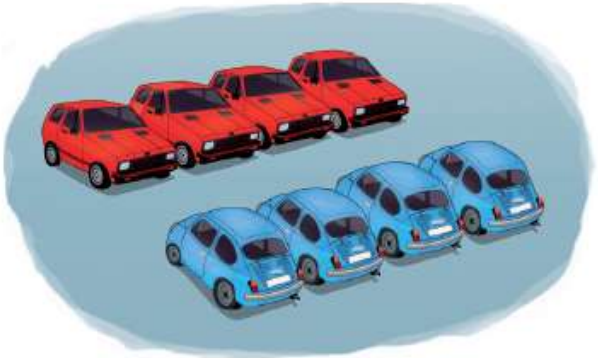


Fig. 1

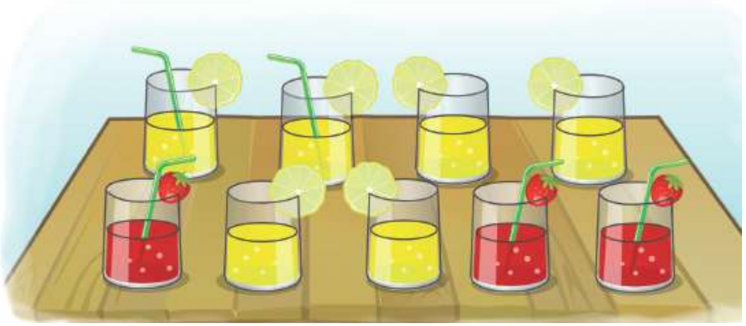


Fig. 2

- Then, on hearing the names of these elements, children select the sets *A* and *B* together with their union *AUB*. This selection is a mental operation provoked by questions:

How many orange juices are there?

How many strawberry juices are there?

How many (juices) alltogether are there?

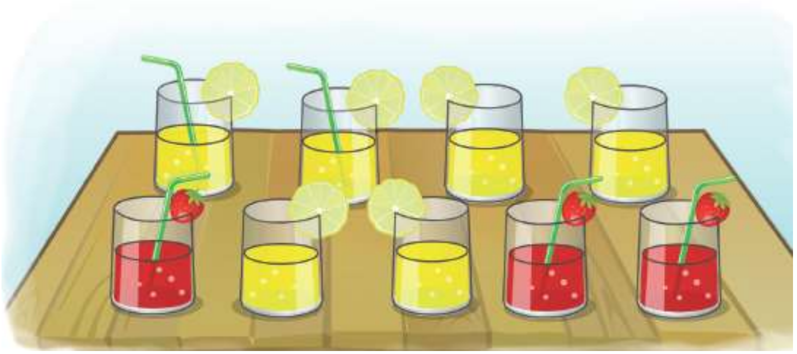
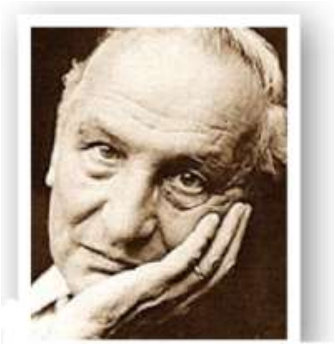


Fig.3

- All numbers are found by counting and all answers are given orally.

NUMBERS AS VISIBLE SHAPES



- *Numbers are visible shapes* says R. Arnheim in his book “Visual Thinking”.
- This is really true for all numbers from the initial blocks.
- Also, in didactics, several systems of number pictures are used to represent visually numbers up to ten.

Fig. 4

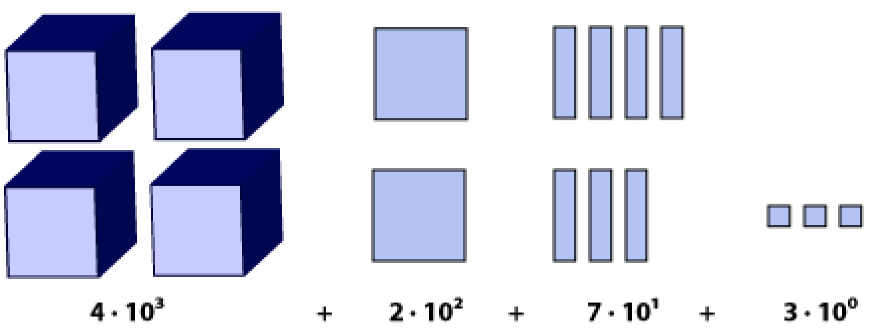


Fig.5

Numbers as visible shapes

- In ancient civilizations numbers were represented (what also means denoted) in the form of specific arrangements of uniform signs.
- For example, in Babylon, by cuneiform signs, in Egypt, ones were represented by sticks, tens by arcs, hundreds by "spirals", etc.

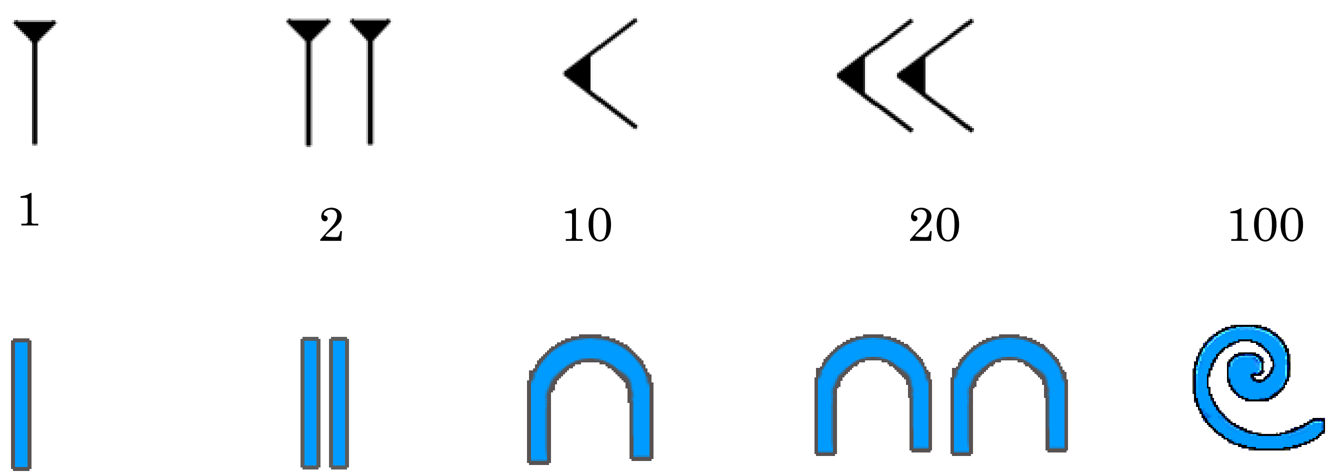


Fig.6

First example of such a system was suggested by Russian philanthropist Busse. (Фёдор Иванович Бусе, 1794 – 1859)

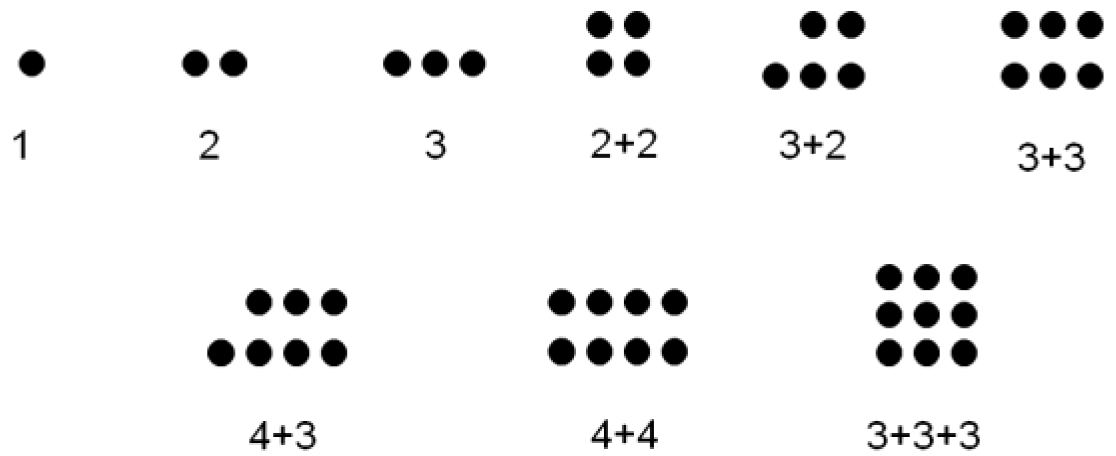


Fig.7

Both, ancient arrangements and those in didactics had to be deliberately structured so that they project the corresponding number at the first glance. Contrary to it, when objects are chaotically grouped and when their number exceeds five, then, without counting, it is usually hard to tell their exact number.

NUMBERS AS VISIBLE SHAPES

A very suggestive representation of numbers is seen on domino pieces, where the spots are arranged into easily remembered shapes which immediately project the number values.

The following pictures represent numbers up to 5 and

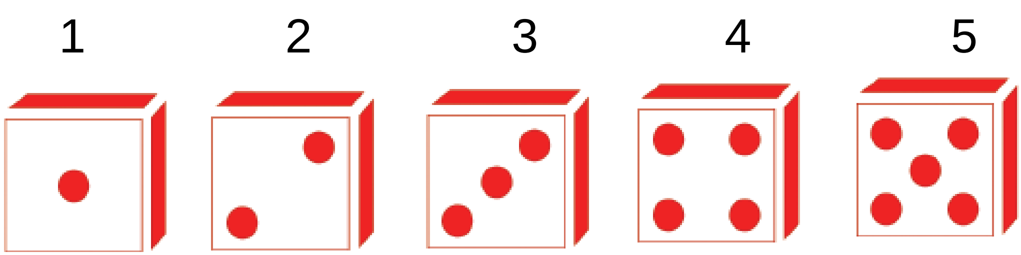


Fig.8

relations between them.

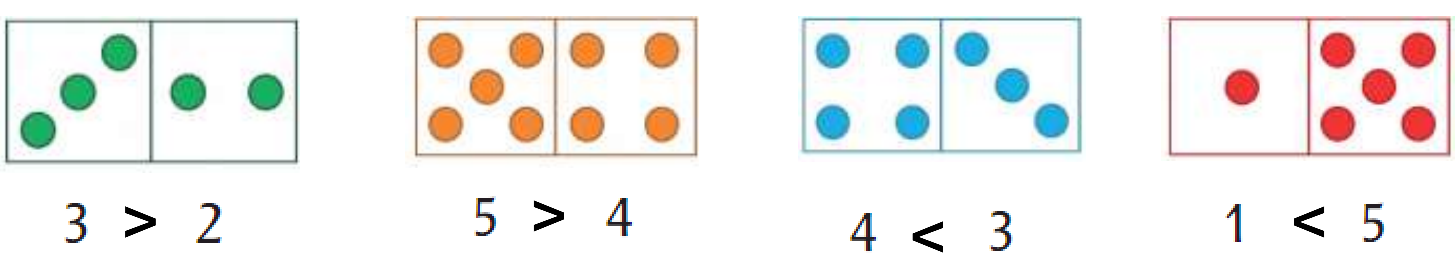


Fig.9

These pictures illustrate addition of two and three numbers.

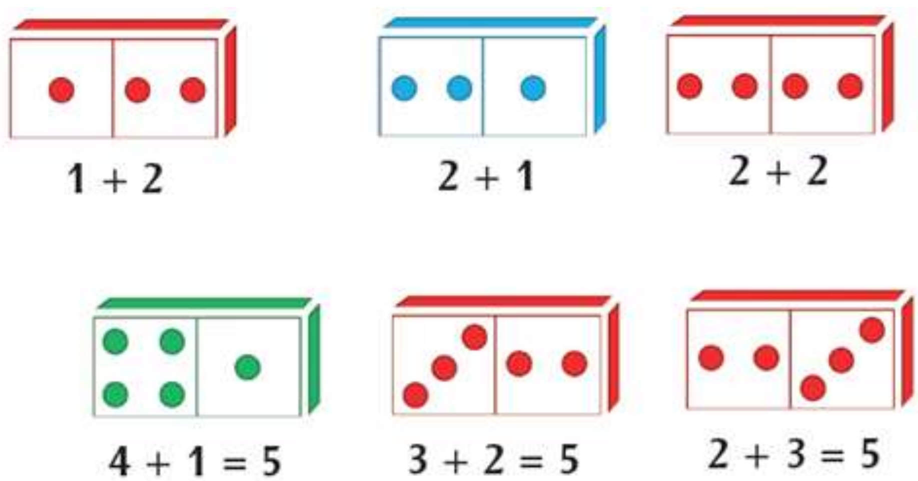


Fig.10

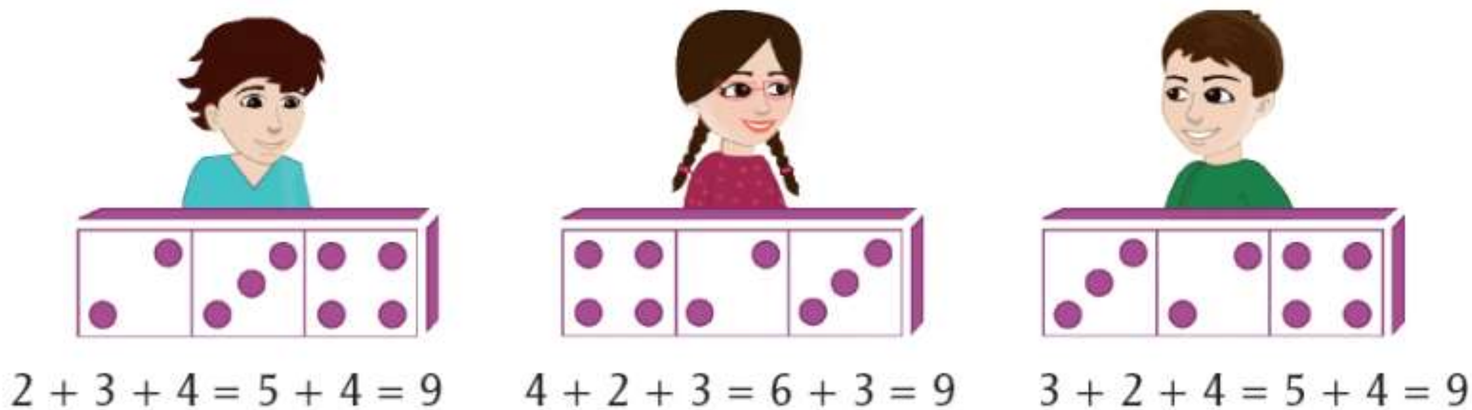


Fig.11

Numbers are visible shapes

- We will confine our considerations to number pictures as the way of visible representation of numbers and operations.
- This system of representing numbers will be called the *arrangements of circlets.*

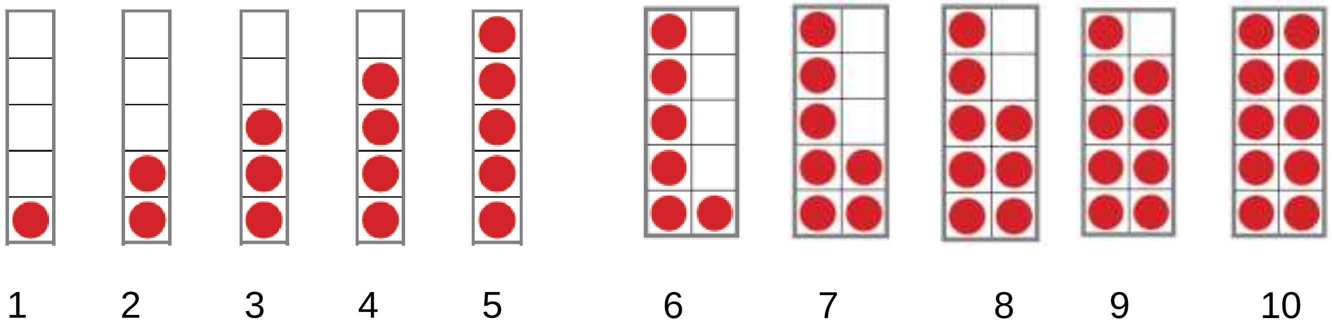


Fig.12

Numbers are visible shapes

- Such a way of representing is also feasible in case of numbers belonging to larger number blocks and it can be carried out easily using a variety of printed material and by means of computer animation.
- We can also say:

  when less noise is present - learning effects are better

  and when the objective is development of calculation skills:

  once chosen the way of representing numbers should not be changed and it should be applied throughout all didactical procedures.

NUMBERS ARE VISIBLE SHAPES

Each way of visible representing of numbers and operations should meet the following requirements:

1. The unit of counting is always represented by the same sign.

This means that number pictures or, as we will often call them, arrangements of signs are patterns composed of uniform signs: bold dots, circlets, the straight line segments of the same length, etc.

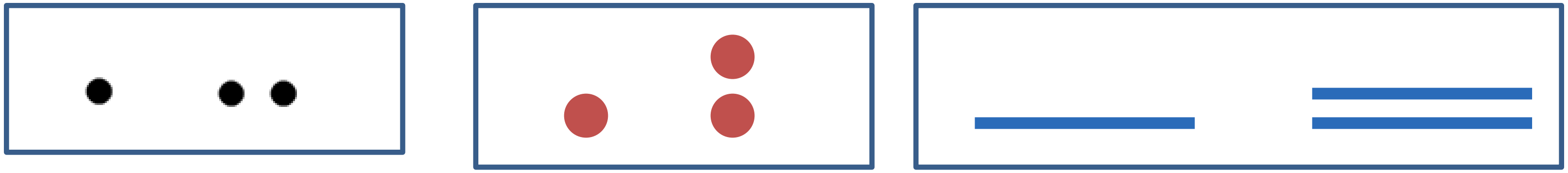


Fig.13

2. The way of arranging respects counting by one and emphasizes the role of numbers 5 and 10.

To respect counting means that when an arrangement represents a certain number, by adding or removing one sign, the new arrangement will represent the following number or the preceding one, respectively.

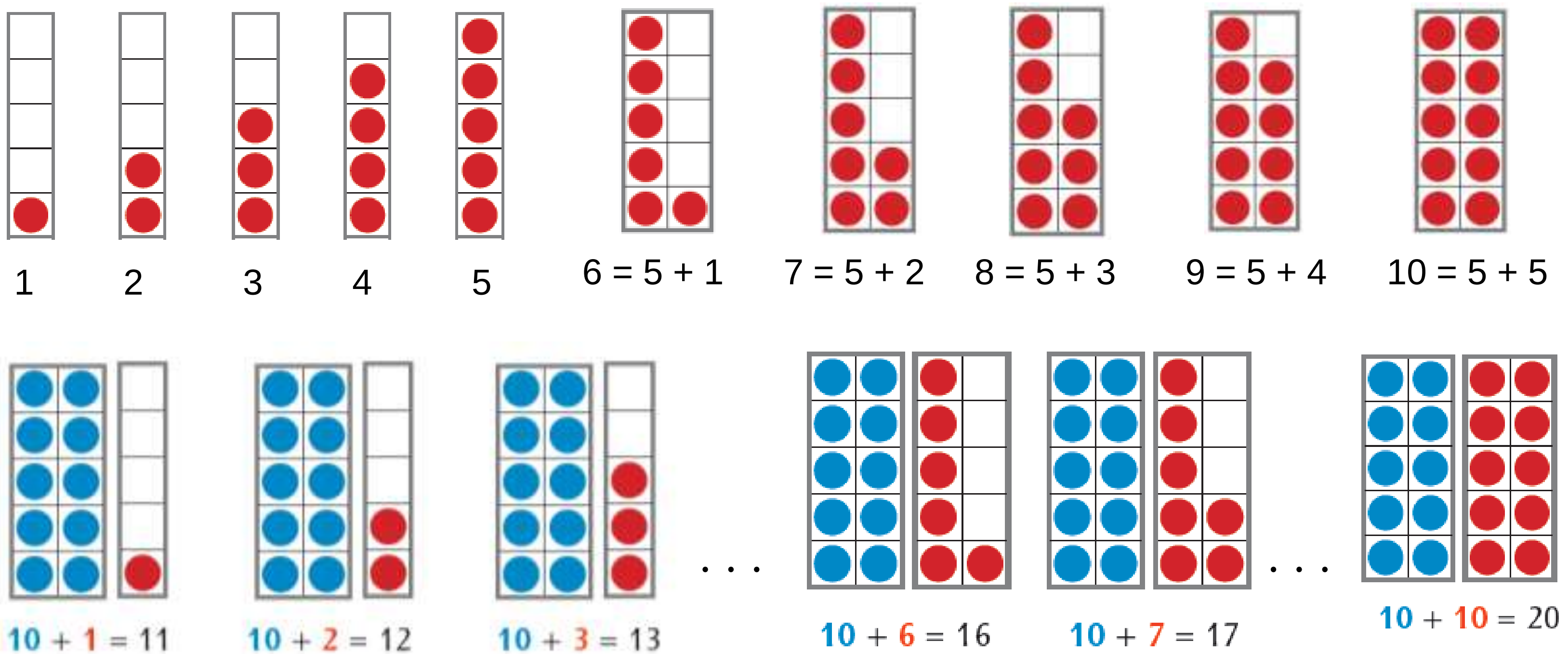
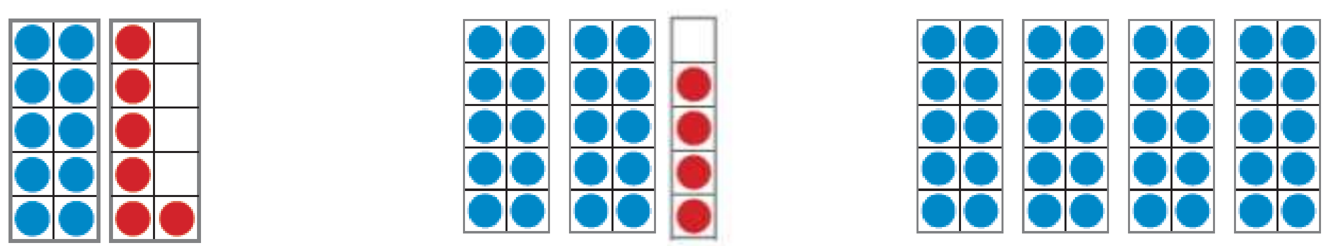


Fig.14

3. Arrangements project number values at the first glance.

This means that arrangements are easily recognizable shapes having a distinct structure which is in accordance with decimal notation and the place of numbers 5 and 10. Their didactical role is to be visual notations for the numbers similar to those used in ancient civilizations.



10 + 6 = 16   20 + 4 = 24   40

Fig.15

4. The shape of arrangements is sufficiently stable.

When the signs representing the unit are drawn with some irregularities, (their size is not even and their places in an arrangement are disordered), then such a pattern may look chaotic and lose the function of being quickly recognizable.

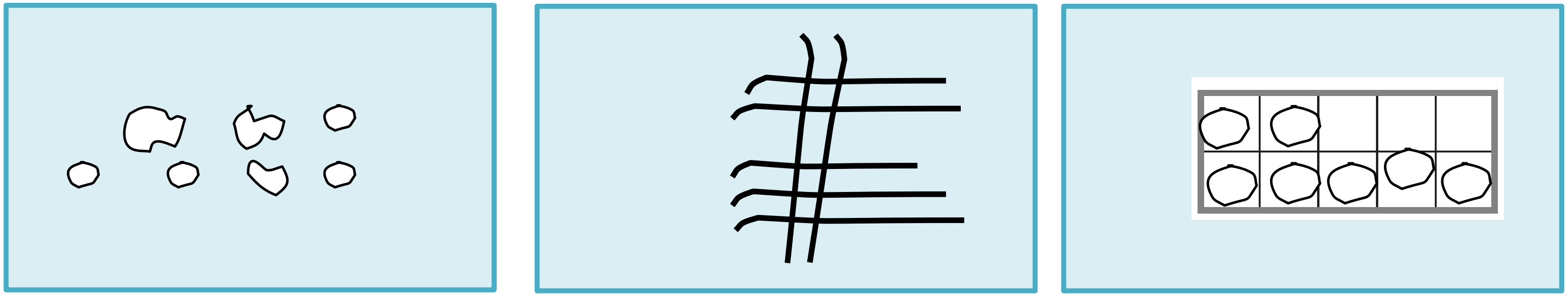


Fig.16

5. Even children can draw arrangements easily (or complete them).

It is purposeless to force children to draw arrangements representing larger numbers. Such numbers should be represented by pictures already existing in the printed materials. But in the case of smaller numbers (up to 20) some drawing activities are desirable. In case of arrangements of circles a matrix of squares should be attached.

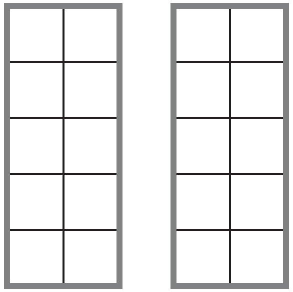


Fig.17

6. Arrangements can be used for representing numbers from larger blocks

Next pictures represent addition and subtraction using number pictures.

Fig.18

$25 + 2 = 20 + (5 + 2) = 20 + 7 = 27$

Units of the two summands are in red,

Fig.19

$38 - 3 = 30 + (8 - 3) = 30 + 5 = 35$

subtrahend is in red

At the end, let us say that the understanding of arrangements as means of calculation is naïve. Their didactical purpose is different – they exist to represent numbers as visible shapes and operations as the activity of rearrangement of such shapes.

Thank you!

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